

Finite time thermodynamic evaluation of endoreversible Stirling heat engine at maximum power conditions

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ABSTRACT

Maximum power and efficiency at the maximum power point of an endoreversible Stirling heat engine with finite heat capacitance rate of external fluids in the heat source/sink reservoirs with regenerative losses are treated. It was found that the thermal efficiency depends on the regenerator effectiveness and the internal irreversibility resulting from the working fluid for a given value of reservoir temperature. It was also concluded that it is desirable to have larger heat capacity of the heat sink in comparison to the heat source reservoir for higher maximum power output and lower heat input.

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1. Introduction

The rapid depletion of our natural resources has attracted attention to the emergency to seek new energy sources and rely on effective and innovative means of energy preservation. Among these tools, the Stirling engine [1,2] is considered to be a promoting and advantageous mean of energy conservation. In fact, the Stirling heat engine was first patented in 1816 by Robert Stirling. Since then, several Stirling engines based on his invention have been

built in many forms and sizes [3]. It is defined to be an externally heated, environmentally very clean engine having high theoretical cycle efficiency. Besides, it works with a closed cycle and uses several gases as working fluid such as helium and hydrogen which lead to higher performance. To energize the engine, many energy sources, such as, combustible materials, solar radiation, geothermal hot water, radioisotope energy and so on, can be used. However, the use of these engines did not prove to be successful due to relatively poor material technologies available at that time in addition to the fact that Stirling engine presents great complexity due to the oscillatory character of the working fluid evolutions.

As, the world community has become much more environmentally conscious, further attention to these engines has been again

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Nomenclature

A	area, m^2
C	heat capacitance, kW/K
n	number of moles
N	number of heat transfer units
P	power output, kW
Q	heat, kJ
S	entropy, J/K
T	temperature, K
τ	time, s
U	overall heat transfer coefficient, W/K m^2
V	volume of the working fluid, m^3
W	output work, kJ

Subscripts

C	heat sink
H	heat source
in	inlet
out	outlet
max	maximum
min	minimum
R	regenerator
SC	cold sink
SH	hot source
1, 2, 3, 4	state points

Greek

β	proportionality constant, K/s
η	thermal efficiency
ε	effectiveness

received because they are inherently clean and thermally more efficient. Moreover, as a result of advances in material technology, these engines are currently being considered for variety of applications due to their many advantages like low noise, less pollution and their flexibility as an external combustion engine to utilize a variety of energy sources or fuels. These engines are also under research and development for their use as heat pumps, replacing systems that are not ecofriendly and environmentally acceptable. Nowadays, the popularity of these engines is growing rapidly.

In recent years, the work on these heat engines has been carried out in detail by earlier workers [4–6] following Curzon–Ahlborn [7] analysis.

One of the most impressive results of the Curzon–Ahlborn (CA) paper was that, with an elegant hypothesis, these authors found very reasonable numerical results for the efficiencies of certain power plants. The literature of finite time thermodynamics began with the novel work of Curzon and Ahlborn and previously Novikov [8], they found that the engine, operating at the maximum power regime, has an efficiency given by $\eta_m = 1 - \sqrt{T_H/T_L}$.

The development of finite-time thermodynamics [9–19], a new discipline in modern thermodynamics, provides a powerful tool for performance analysis of practical engineering cycles. Several authors have studied the finite-time thermodynamic performances of the Stirling engine. Many investigators have studied the effect of heat transfer on the power output of a Stirling engine.

Wu et al. [20] showed the effects of heat transfer, regeneration time, and imperfect regeneration on the performance of the irreversible Stirling engine cycle. Erbay and Yavuz [21] analyzed the real Stirling heat engine for maximum power output conditions using polytropic processes. They also determined the efficiency and compression ratio at maximum power density and ascertained the thermal design bounds. Goktun [22] investigated the effect of the

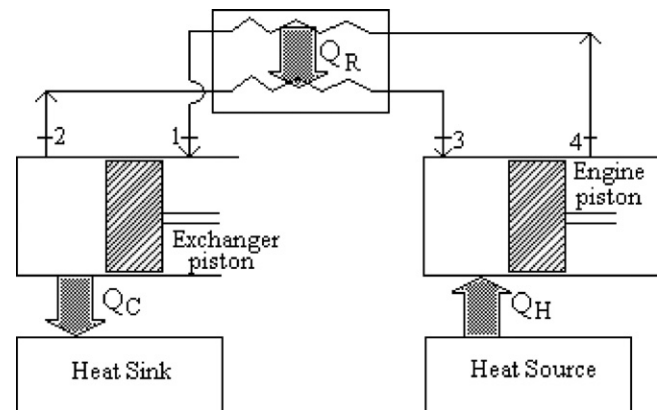


Fig. 1. Schematic equipment for the Stirling cycle.

irreversibility parameter on the performance of a solar heat engine. Maximum power and efficiency at the maximum power point were treated while optimization of the overall system efficiency was considered to be beyond the scope of his study. Solar endoreversible heat engines were investigated by Gordon [23] considering two separate cases of heat loss; linear and radiative. He determined the optimal hot reservoir temperature values, based on fundamental limiting expressions for heat engine and solar collector efficiencies. Wu and Blank [24] examined the power optimization of an extra-terrestrial solar radiant Stirling heat engine with an ideal/perfect regenerator and infinite heat capacity of external heat (source/sink) reservoirs. They showed that the optimal power and the corresponding thermal efficiency are based on higher and lower temperature bounds. Wang et al. [10] have established an irreversible cycle model of the Stirling refrigerator using a ferroelectric material as the working substance, based on a general expression of the polarization of ferroelectric materials and a linear heat-transfer law. They examined influences of irreversibilities on the performance of the ferroelectric Stirling refrigeration cycle. Yan and Chen [25] studied the optimal performance of an endoreversible cycle operating between a finite heat source and sink. Senft [26] studied the theoretical limitations on the performance of a Stirling engine subject to limited heat transfer, external thermal and mechanical losses. After a sequence of fundamental power-maximization problems Bejan [27] concluded that the power output of various power plant configurations can be maximized by properly dividing the fixed inventory of heat exchange equipment among the heat transfer components of each plant.

In this paper, we describe a study on the effect of each of the parameters, i.e. the operating temperatures, effectiveness of each heat exchanger, heat capacitance rate of heat source/sink external fluids on power output and thermal efficiency of an endoreversible Stirling heat engine using finite-time thermodynamics. The maximum power output, the corresponding thermal efficiency of the cycle and the regenerative heat exchanger on the heat transfer (Q_H and Q_C) to and from the heat engine and the regenerative heat transfer (Q_R) have all been analyzed.

2. System description

Heat engines are designed to operate between two limits. One limit is the maximum efficiency operation (reversible limit) which is usually of little practical interest since it corresponds to zero power. The other limit is maximum power operation (irreversible limit) which is often a design objective for Stirling heat engines.

An endoreversible Stirling cycle is depicted in Figs. 1 and 2. This cycle approximates the compression stroke of the real engine as an isothermal process 1–2, with irreversible heat rejection to

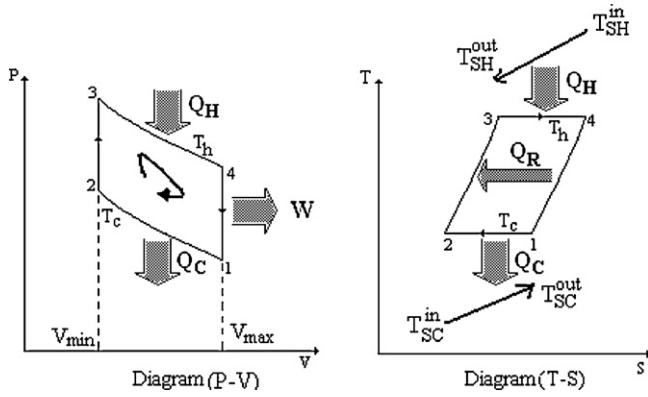


Fig. 2. Stirling engine cycle in a P - V and T - S diagram.

an intermediate low-temperature sink. The heat addition to the working fluid from the regenerator is modeled as a reversible constant-volume process 2–3. The work-producing expansion stroke is modeled as an isothermal process 3–4, with irreversible heat addition from an intermediate high-temperature source. Finally, heat rejection to the regenerator is modeled using a reversible constant-volume process 4–1.

Within real Stirling engines, the external heat-transfer processes 1–2 and 3–4 must each occur in finite times. This in turn requires that these heat-transfer processes must proceed through finite-temperature differences and are therefore defined as being externally irreversible. During the first of these, the heat rejection process 1–2, heat flows from the working fluid which is maintained at a constant temperature T_c , to the intermediate low temperature sink, thus flowing across the temperature difference $(T_{SC}^{in} - T_{SC}^{out})$, as shown in Fig. 1. Similarly, during the heat addition process 3–4, heat is transferred from the intermediate high temperature sources to the working fluid which is maintained at a constant temperature T_h , thus flowing across the temperature difference $(T_{SH}^{in} - T_{SH}^{out})$.

If the regeneration is assumed to be ideal, the heat rejected to the regenerator by the hot working fluid during the process 4–1 is equivalent to the heat supplied to the cold working fluid during 2–3. This can be visualized by noting that the area under the process path on the temperature–entropy diagram in Fig. 2 for process 4–1 area is equal in magnitude to the area under process path 2–3.

3. Finite time thermodynamic analysis

Finite time thermodynamics deal with the fact that there must be a finite temperature difference between the working fluid and the source/sink heat reservoirs in order to transfer a finite amount of heat in a finite time.

The Stirling engine cycle is examined here, assuming limited heat exchanger areas, finite heat transfer coefficients and a defined rotation frequency, which induces speed-dependent fluid flows and time-dependent processes.

The working substance in the Stirling cycle is assumed to be an ideal gas, the Stirling cycle consists of two isothermal and two isochoric processes, as shown in Fig. 2 in which Q_h and Q_c are, respectively, the amounts of heat absorbed from the heat source at temperature T_h and released to the heat sink at temperature T_c by the working substance during two isothermal processes, V_{max} and V_{min} are the volumes of the working substance during the regenerative (constant volume) processes 4–1 and 2–3, respectively, the amount of heat absorbed Q_h and the heat released Q_c can be given as:

$$Q_h = T_h \Delta S_{3-4} = nRT_h \left(\ln \frac{V_{max}}{V_{min}} \right) \quad (1)$$

$$Q_c = T_c \Delta S_{1-2} = nRT_c \left(\ln \frac{V_{max}}{V_{min}} \right) \quad (2)$$

where R is the universal gas constant and n is the number of moles of the working substance.

The effect of finite heat transfer on the performance of thermodynamic cycles is considered in all exchanger regenerator, cooler and heater. So the heat transfer rate by heater and cooler are respectively:

$$Q_h = U_h A_h \Delta T_{\log}^h \tau_{3-4} = C_h (T_{SH}^{in} - T_{SH}^{out}) \tau_{3-4} \quad (3)$$

$$Q_c = U_c A_c \Delta T_{\log}^c \tau_{1-2} = C_c (T_{SC}^{in} - T_{SC}^{out}) \tau_{1-2} \quad (4)$$

where τ_{3-4} and τ_{1-2} are heat addition and heat rejection times for the Stirling heat engine, $U_h A_h$ and $U_c A_c$ are the overall heat transfer coefficient and area products for hot and cold sides, respectively. C_h and C_c are, respectively, the heat capacitance rates of external fluids in heat source/sink reservoirs.

With $\Delta T_{\log}^h = ((\Delta T_a - \Delta T_b) / (\ln(\Delta T_a / \Delta T_b)))$ as the mean logarithmic temperature, where ΔT_a and ΔT_b are the temperature differences between the two fluids at each end of the exchanger $\Delta T_a = T_{SH}^{in} - T_h$ and $\Delta T_b = T_{SH}^{out} - T_h$

$\Delta T_{\log}^c = ((\Delta T_e - \Delta T_f) / (\ln(\Delta T_e / \Delta T_f)))$ as the mean logarithmic temperature, where ΔT_e and ΔT_f are the temperature differences between the two fluids at each end of the exchanger $\Delta T_e = T_c - T_{SC}^{in}$ and $\Delta T_f = T_c - T_{SC}^{out}$.

The hot-end heat exchanger effectiveness is defined as:

$$\varepsilon_H = \frac{T_{SH}^{in} - T_{SH}^{out}}{T_{SH}^{in} - T_h} \quad (5)$$

For the cold-end heat exchanger effectiveness is:

$$\varepsilon_C = \frac{T_{SC}^{out} - T_{SC}^{in}}{T_c - T_{SC}^{in}} \quad (6)$$

where N_h and N_c are respectively the number of heat transfer units, $N_h = ((U_h A_h) / C_h)$ and $N_c = ((U_c A_c) / C_c)$.

So ε_H and ε_C are respectively given as:

$$\varepsilon_H = 1 - \exp \left(-\frac{U_h A_h}{C_h} \right) = 1 - e^{-N_h} \quad (7)$$

$$\varepsilon_C = 1 - \exp \left(-\frac{U_c A_c}{C_c} \right) = 1 - e^{-N_c} \quad (8)$$

Using Eqs. (1)–(6) we obtain:

$$Q_h = C_h \varepsilon_H (T_{SH}^{in} - T_h) \tau_{3-4} \quad (9)$$

$$Q_c = C_c \varepsilon_C (T_c - T_{SC}^{in}) \tau_{1-2} \quad (10)$$

It is also crucial to consider the finite heat transfer through the regenerator. Let ΔQ_R is the regenerative heat loss per cycle during the two regeneration processes, which is proportional to the temperature difference of the working fluid therefore the regenerative heat transfer is given by [3–8]:

$$Q_R = n C_v \varepsilon_R (T_h - T_c) \quad (11)$$

where C_v is the specific heat of the working substance and ε_R is the effectiveness of the regenerator. Thus,

$$\Delta Q_R = n C_v (1 - \varepsilon_R) (T_h - T_c) \quad (12)$$

When all the irreversibilities, mentioned above are taken into account, the net amount of heat absorbed from the source and released to the sink are given as:

$$Q_H = Q_h + \Delta Q_R \quad (13)$$

$$Q_C = Q_c + \Delta Q_R \quad (14)$$

Owing to the influence of irreversibility due to finite heat transfer, the regenerative processes need a non-negligible time compared with the time of the two isothermal processes. In order to calculate the time of the regenerative processes, it is assumed that the temperature of the working substance in the regenerative processes as a function of time is given by [4–11], and is proportional to the temperature difference of the working fluid.

$$\frac{dT}{dt} = \pm\beta \quad (15)$$

where β is the proportionality constant and is independent of the temperature difference but depends on the property of regenerative material.

The time of two regenerative processes is:

$$\tau_{2-3} = \tau_{4-1} = \frac{T_h - T_c}{\beta}$$

$$\text{So } \tau_R = \tau_{2-3} + \tau_{4-1} = \frac{2(T_h - T_c)}{\beta} \quad (16)$$

where τ_{2-3} and τ_{4-1} are the time taken during two regenerative processes 2–3 and 4–1, respectively.

Thus the total cycle time is given by:

$$\tau_{\text{cyc}} = \tau_R + \tau_{1-2} + \tau_{3-4} \quad (17)$$

From the first-law of thermodynamics we have:

$$W = Q_H - Q_C = Q_h - Q_c \quad (18)$$

Thus, the power output and the corresponding thermal efficiency will be:

$$P = \frac{W}{\tau_{\text{cyc}}} = \frac{Q_h - Q_c}{\tau_R + \tau_{1-2} + \tau_{3-4}} \quad (19)$$

$$\eta = \frac{W}{Q_H} = \frac{Q_h - Q_c}{Q_h + \Delta Q_R} \quad (20)$$

Using Eqs. (1), (2), (9), (10) and (16), we get power output as given by:

Therefore

$$P = \frac{nR \ln(V_{\text{max}}/V_{\text{min}})(T_h - T_c)}{((Q_h/(C_h \varepsilon_h (T_{SH}^{\text{in}} - T_h))) + (Q_c/(C_c \varepsilon_c (T_c - T_{SC}^{\text{in}}))) + (2(T_h - T_c)/\beta))}$$

$$\text{Therefore } P = \frac{nR \ln(V_{\text{max}}/V_{\text{min}})(T_h - T_c)}{((nR \ln(V_{\text{max}}/V_{\text{min}})T_h)/(C_h \varepsilon_h (T_{SH}^{\text{in}} - T_h))) + ((nR \ln(V_{\text{max}}/V_{\text{min}})T_c)/(C_c \varepsilon_c (T_c - T_{SC}^{\text{in}}))) + (2(T_h - T_c)/\beta)}$$

$$\text{So } P = \frac{(T_h - T_c)}{(T_h/(C_h \varepsilon_h (T_{SH}^{\text{in}} - T_h))) + (T_c/(C_c \varepsilon_c (T_c - T_{SC}^{\text{in}}))) + (2(T_h - T_c)/\beta nR \ln(V_{\text{max}}/V_{\text{min}}))} \quad (21)$$

And the thermal efficiency η is given by:

$$\eta = \frac{W}{Q_H} = \frac{Q_h - Q_c}{Q_h + \Delta Q_R} = \frac{nR \ln(V_{\text{max}}/V_{\text{min}})(T_h - T_c)}{nR \ln(V_{\text{max}}/V_{\text{min}})T_h + nC_v(1 - \varepsilon_R)(T_h - T_c)}$$

$$\text{So } \eta = \frac{T_h - T_c}{T_h + (T_h - T_c)C_v(1 - \varepsilon_R)/R \ln(V_{\text{max}}/V_{\text{min}})} \quad (22)$$

We see that the output power and the corresponding thermal efficiency are functions of two variable T_h and T_c for given parameters like T_{SH}^{in} , T_{SC}^{in} , ε_H , ε_C , C_h , C_c etc. Thus maximizing power P according to T_h and T_c , we have $\partial P/\partial T_h = 0$ and $\partial P/\partial T_c = 0$ which yields optimal values of T_h and T_c as given by:

$$T_h = \sqrt{\frac{T_{SH}^{\text{in}} T_{SC}^{\text{in}} (1 + \sqrt{C_h \varepsilon_h T_{SH}^{\text{in}}/C_c \varepsilon_c T_{SC}^{\text{in}}})}{T_{SC}^{\text{in}} (1 + \sqrt{C_h \varepsilon_h/C_c \varepsilon_c})}} \quad (23)$$

$$T_c = \frac{T_{SC}^{\text{in}} (1 + \sqrt{C_h \varepsilon_h T_{SH}^{\text{in}}/C_c \varepsilon_c T_{SC}^{\text{in}}})}{1 + \sqrt{C_h \varepsilon_h/C_c \varepsilon_c}} \quad (24)$$

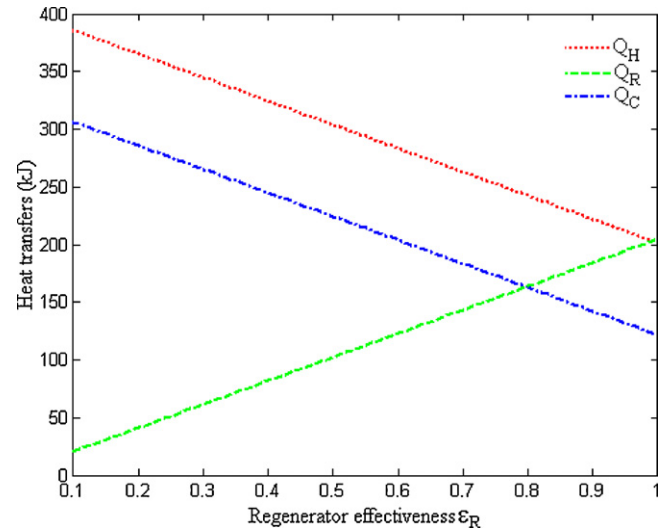


Fig. 3. Effect of ε_R on heat transfer ($\varepsilon_C = \varepsilon_H = 0.7$, $C_H = C_C = 1$ kW/K, $V_1/V_2 = 3.5$).

Substituting the values of T_h and T_c into Eqs. (21) and (22), the maximum power output is given by:

$$P_{\text{max}} = \frac{K(\sqrt{T_{SH}^{\text{in}}} - \sqrt{T_{SC}^{\text{in}}})^2}{1 + Ka(\sqrt{T_{SH}^{\text{in}}} - \sqrt{T_{SC}^{\text{in}}})^2} \quad (25)$$

And the corresponding thermal efficiency at maximum power will be:

$$\eta_{\text{max}} = \frac{(1 - \sqrt{T_{SC}^{\text{in}}/T_{SH}^{\text{in}}})}{1 + b(1 - \sqrt{T_{SC}^{\text{in}}/T_{SH}^{\text{in}}})} = \frac{\eta_{CA}}{1 + b\eta_{CA}} \quad (26)$$

where $K = C_h \varepsilon_h C_c \varepsilon_c / (\sqrt{C_h \varepsilon_h} + \sqrt{C_c \varepsilon_c})^2$; $a = 2/nR \beta \ln(V_{\text{max}}/V_{\text{min}})$; $b = C_v(1 - \varepsilon_R)/R \ln(V_{\text{max}}/V_{\text{min}})$ and $\eta_{CA} = 1 - \sqrt{T_{SC}^{\text{in}}/T_{SH}^{\text{in}}}$ is the Curzon–Ahlborn efficiency of an endoreversible Carnot heat engine

with corresponding source/sink temperatures. It is seen from Eq. (25) that the maximum power output of the Stirling heat engine is not affected by the regenerative heat loss, although it depends upon the time of regenerative processes. Eq. (26) also clearly shows that when a Stirling heat engine with regenerative loss is operating in the maximum power output, its efficiency is different from that of an endoreversible Carnot heat engine efficiency η_{CA} .

4. Results and discussions

In order to study the effect of the effectiveness of each heat exchanger and the regenerator, the volume ratio $V_{\text{max}}/V_{\text{min}}$, the inlet source and sink temperatures T_{SH}^{in} and T_{SC}^{in} and the heat capacitance rates of the heat source/sink reservoirs on the heat transfers Q_H and Q_C to and from the heat engine, the regenerative heat transfer Q_R , the output work W , the maximum power output P_{max} and the corresponding thermal efficiency of the Stirling heat engine, we have developed a computation procedure, described on Matlab

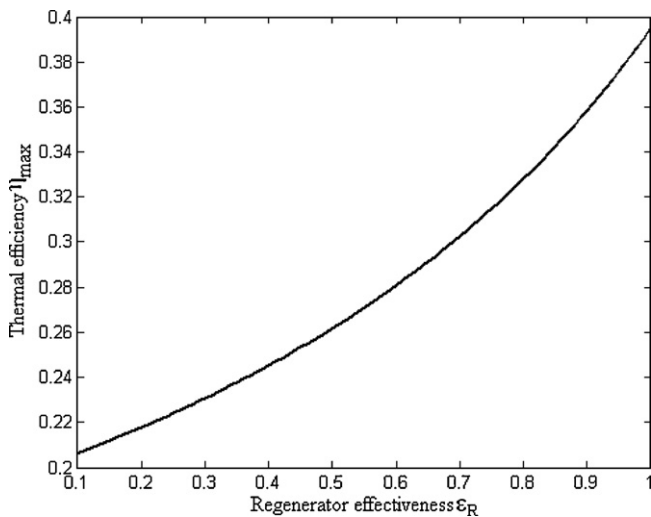


Fig. 4. Effect of ε_R on thermal efficiency ($\varepsilon_C = \varepsilon_H = 0.7$, $C_H = C_C = 1 \text{ kW/K}$, $V_1/V_2 = 3.5$).

program, for an endoreversible Stirling heat engine with an imperfect regeneration. The result discussion is given below.

4.1. Effect of the regenerator effectiveness ε_R

The heat transfers Q_H and Q_C decline whereas the regenerative heat transfer Q_R and the thermal efficiency $\eta_{\max}$ escalate when the regenerator effectiveness ε_R augments, despite the fact that the output work W and the maximum power output $P_{\max}$ persist constant as shown in Figs. 3 and 4. It is also observed that when there is no heat loss through the regenerator $\varepsilon_R = 1$, the Stirling heat engine attains an efficiency equal to the Curzon–Ahlborn efficiency $\eta_{CA} = 39.49$ of an endoreversible Carnot heat engine. The regenerator effectiveness should be high enough from the perspective of higher thermal efficiency as regards as for lower internal irreversibility in the heat engine as shown in Fig. 4.

Nevertheless, an endoreversible Stirling heat engine with an ideal regenerator is as efficient as an endoreversible Carnot heat engine but it is not practical since an ideal regeneration requires infinite time or infinite regenerative area.

Thus we see that for highly effective regenerators (close to $\varepsilon = 1$) a 1% reduction in regenerator effectiveness results in a more than 3% reduction in thermal efficiency $\eta_{\max}$. Furthermore, we deduce that if one has a regenerator with an effectiveness of 0.8, the thermal efficiency will drop to around 30%.

4.2. Effect of the hot side heat exchanger effectiveness ε_H

The thermal efficiency persists constant as shown in Figs. 5 and 6 whereas the heat transfers Q_H and Q_C , the regenerative heat transfer Q_R , the output work W and the maximum power output $P_{\max}$ augment in conjunction with the rise of the hot heat exchanger effectiveness. A point to note is that the heat exchangers affect primarily the maximum power output $P_{\max}$ and not the heat input Q_H to the heat engine.

4.3. Effect of the cold side heat exchanger effectiveness ε_C

When the cold side heat exchanger effectiveness ε_C escalates, the heat transfers Q_H and Q_C , the regenerative heat transfer Q_R and the output work W decline whereas the maximum power output $P_{\max}$ rises, and the thermal efficiency persists constants as demonstrated in Figs. 7 and 8. Thus, the impact of ε_C is more notable for the

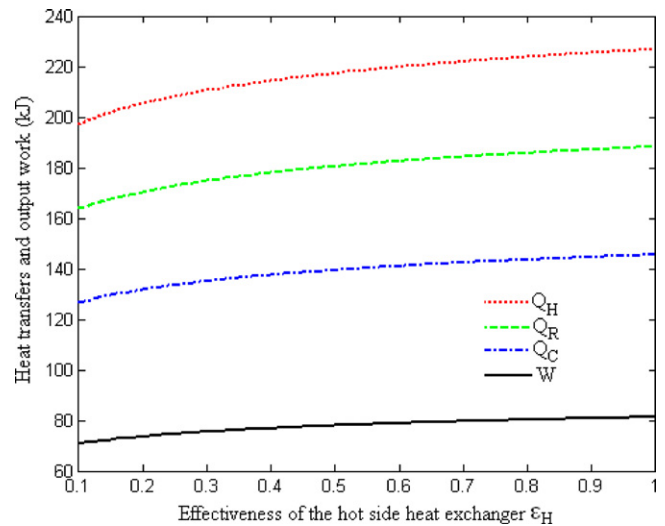


Fig. 5. Effect of ε_H on heat transfer and output work ($\varepsilon_R = 0.9$, $\varepsilon_C = 0.7$, $C_H = C_C = 1 \text{ kW/K}$, $V_1/V_2 = 3.5$, $\eta_{\max} = 35.84\%$).

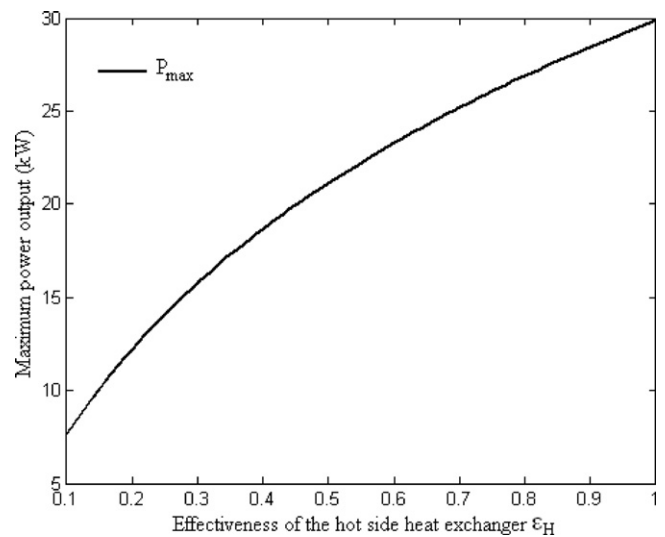


Fig. 6. Effect of ε_H on maximum power output ($\varepsilon_R = 0.9$, $\varepsilon_C = 0.7$, $C_H = C_C = 1 \text{ kW/K}$, $V_{\max}/V_{\min} = 3.5$, $\eta_{\max} = 35.84\%$).

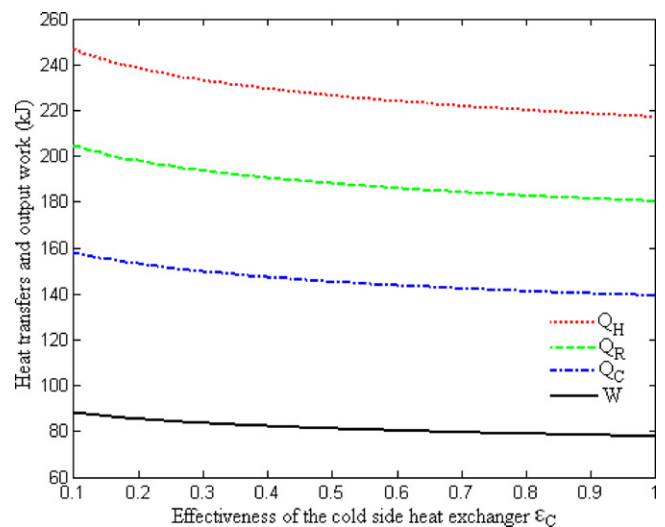


Fig. 7. Effect of ε_C on heat transfer and output work ($\varepsilon_R = 0.9$, $\varepsilon_H = 0.7$, $C_H = C_C = 1 \text{ kW/K}$, $V_1/V_2 = 3.5$, $\eta_{\max} = 35.84\%$).

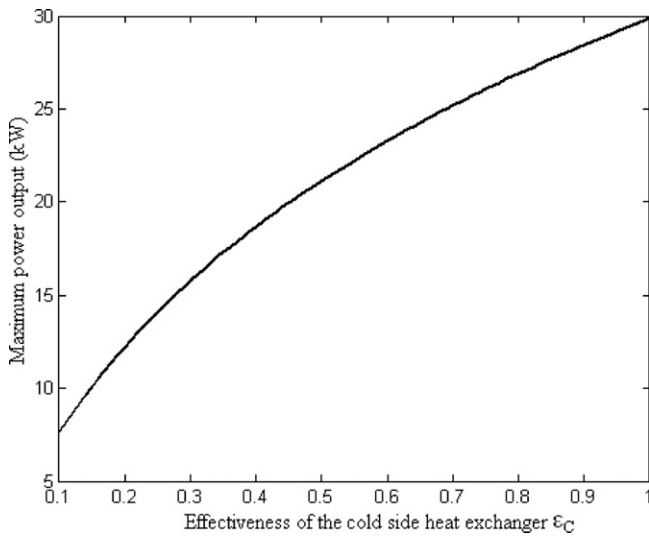


Fig. 8. Effect of ε_H on maximum power output ($\varepsilon_R = 0.9$, $\varepsilon_H = 0.7$, $C_H = C_C = 1$ kW/K, $V_{max}/V_{min} = 3.5$, $\eta_{max} = 35.84\%$).

maximum power output P_{max} and less notable for the heat transfer Q_C from the heat engine.

Thus, we conclude that ε_C and ε_H affect only the maximum power output and not the thermal efficiency contrary to the regenerator effectiveness.

4.4. Effect of the volume ratio V_{max}/V_{min}

When the volume ratio V_{max}/V_{min} increases, the heat transfers Q_H and Q_C and the output work W increase, while the regenerative heat transfer Q_R and the thermal efficiency persist constant as observed in Fig. 9. The effect of volume ratio V_{max}/V_{min} is more notable for the heat input Q_H and less notable for the output work W . Having a maximum volume ratio is required in order to obtain the output work W ; therefore, we deduce that the constant cooling volume should be as higher as possible than the constant heating volume.

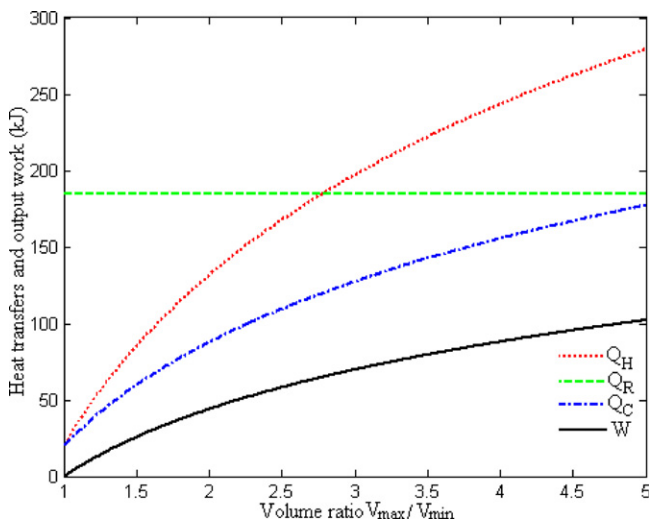


Fig. 9. Effect of the volume ratio on heat transfers and output work ($T_{SH}^{in} = 923$ K, $T_{SC}^{in} = 338$ K, $\varepsilon_R = 0.9$, $\varepsilon_C = \varepsilon_H = 0.7$, $C_C = C_H = 1$ kW/K, $\eta_{max} = 35.84\%$).

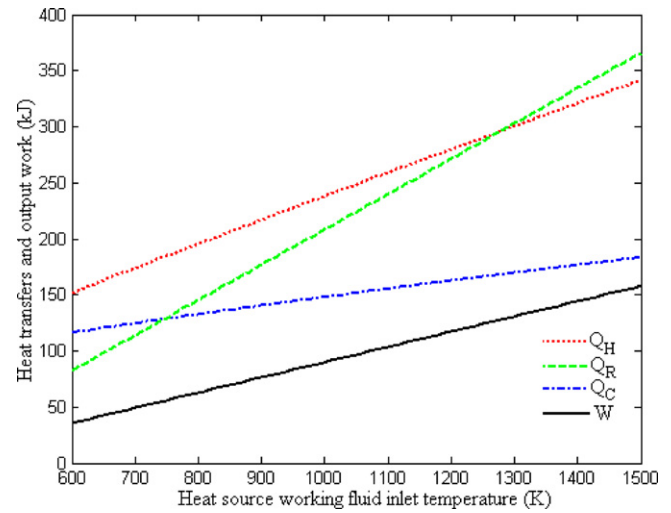


Fig. 10. Effect of the heat source working fluid inlet temperature on heat transfers and output work ($T_{SC}^{in} = 338$ K, $\varepsilon_R = 0.9$, $\varepsilon_C = \varepsilon_H = 0.7$, $C_H = C_C = 1$ kW/K, $V_1/V_2 = 3.5$).

4.5. Effect of the heat source working fluid inlet temperature T_{SH}^{in}

The heat transfers Q_H and Q_C , the regenerative heat transfer Q_R , the output work W , the maximum power output P_{max} and the thermal efficiency η_{max} increase all when the external heat source fluid inlet temperature T_{SH}^{in} increases as illustrated by Figs. 10 and 11. The effect of T_{SH}^{in} is more notable for the maximum power output P_{max} and less notable for the thermal efficiency η_{max} . Thus, having a high temperature heat source from the point of view of higher power output in addition to higher thermal efficiency is really recommended.

4.6. Effect of the heat sink working fluid inlet temperature T_{SC}^{in}

When the heat sink working fluid inlet temperature T_{SC}^{in} rises, the heat transfers Q_H and Q_C increase, while the regenerative heat transfer Q_R , the output work W , the maximum power output P_{max} and the thermal efficiency η_{max} decrease as shown in Figs. 12 and 13. The impact of T_{SC}^{in} is more noticeable for the maximum power output P_{max} and less noticeable for the heat input Q_H to the heat engine. Therefore, we would rather have a

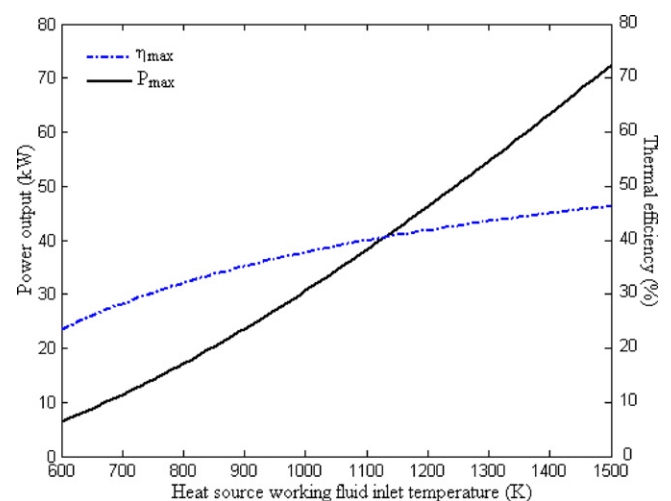


Fig. 11. Effect of the heat source working fluid inlet temperature on power output and thermal efficiency ($T_{SC}^{in} = 338$ K, $\varepsilon_R = 0.9$, $\varepsilon_C = \varepsilon_H = 0.7$, $C_H = C_C = 1$ kW/K, $V_1/V_2 = 3.5$).

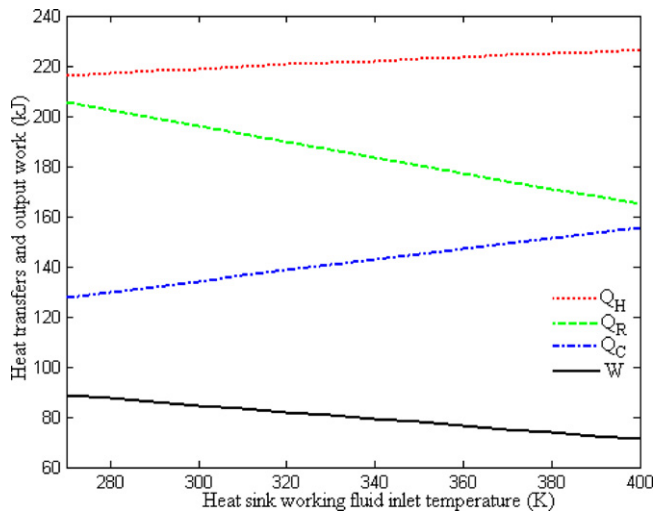


Fig. 12. Effect of the heat sink working fluid inlet temperature on heat transfers and output work ($T_{SH}^{in} = 923$ K, $\varepsilon_R = 0.9$, $\varepsilon_C = \varepsilon_H = 0.7$, $C_H = C_C = 1$ kW/K, $V_1/V_2 = 3.5$).

low temperature heat sink to obtain higher power output P_{max} as well as higher thermal efficiency η_{max} .

4.7. Effect of the heat capacitance rate of the heat source C_H

The heat transfers Q_H and Q_C , the regenerative heat transfer Q_R , the output work W , and the maximum power output P_{max} rise in conjunction with the increase of the heat capacitance rate C_H of the heat source reservoir, while the thermal efficiency persists constant as illustrated in Figs. 14 and 15. Therefore, the impact of C_H is more noticeable for the maximum power output P_{max} and less noticeable for the regenerative heat transfer Q_R .

4.8. Effect of the heat capacitance rate of the heat sink C_C

When the heat capacitance rate of the heat sink C_C reservoir rises, the heat transfers Q_H and Q_C , the regenerative heat transfer Q_R and the output work W decline while the maximum power output P_{max} escalates, whereas the thermal efficiency persists constant as shown in Figs. 16 and 17. The effect of C_C is more notable for the maximum power P_{max} output and less notable for the regenerative heat transfer Q_R .

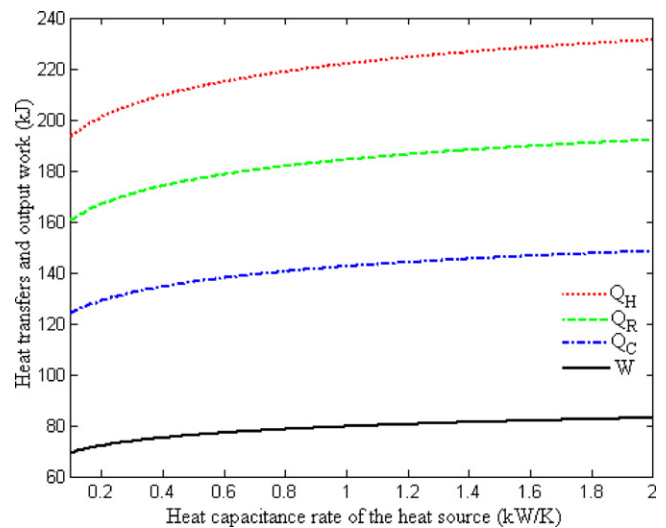


Fig. 14. Effect of the heat capacitance rate of the heat source on heat transfers and output work ($T_{SH}^{in} = 923$ K, $T_{SH}^{in} = 338$ K, $\varepsilon_R = 0.9$, $\varepsilon_C = \varepsilon_H = 0.7$, $C_C = 1$ kW/K, $\eta_{max} = 35.84\%$, $V_1/V_2 = 3.5$).

We can also notice from these figures that when C_H increases, both Q_H and Q_C augment whereas when C_C increases, both Q_H and Q_C decline. While the power output remains always the same in both conditions. We deduce from this observations that while increasing C_C , we must provide less heat to the engine in order to obtain the same power output as one would gain it by rising C_H with more heat input. Thus, it is recommended to have higher C_C rather than C_H from the perspective of lower heat transfers to and from the heat engine as well as higher power output. Instead, irreversibilities produced on the engine source/sink sides can justify this fact.

By increasing C_H , the total irreversibilities rise whereas when C_C increases the total irreversibilities decline. Although C_H or C_C did not have an effect on the internal irreversibility secondary to regenerative heat losses.

The source side external irreversibility decreases whereas it increases on the sink as C_H increases. Conversely, external irreversibility increases on the source side whereas on the sink side it decreases as C_C rises. Therefore, having higher C_C and lower C_H are recommended.

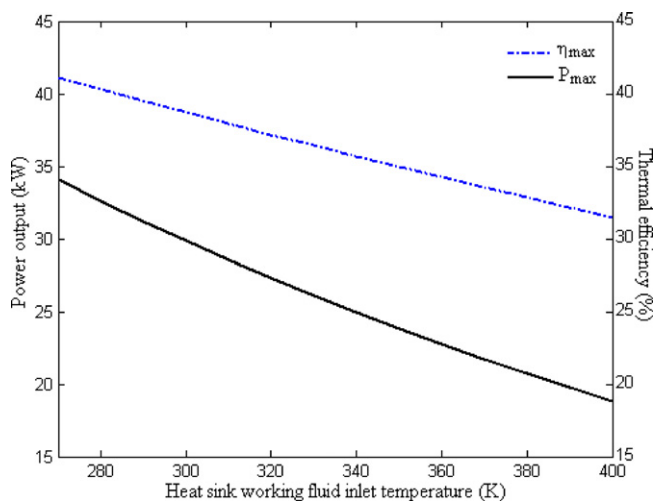


Fig. 13. Effect of the heat sink working fluid inlet temperature on power output and thermal efficiency ($T_{SH}^{in} = 923$ K, $\varepsilon_R = 0.9$, $\varepsilon_C = \varepsilon_H = 0.7$, $C_H = C_C = 1$ kW/K, $V_1/V_2 = 3.5$).

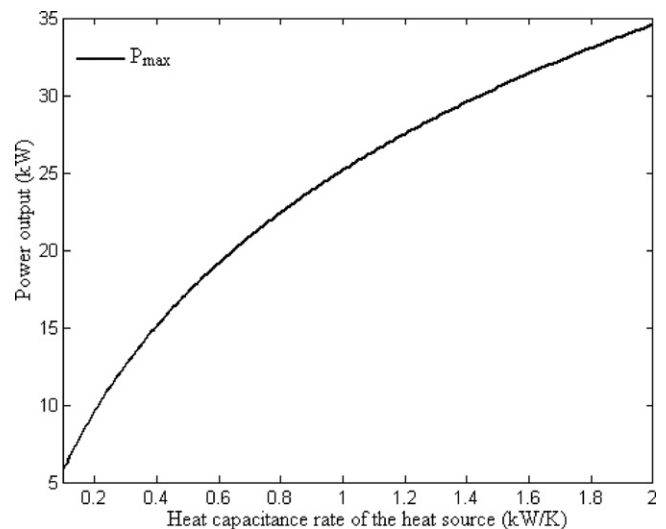


Fig. 15. Effect of the heat capacitance rate of the heat source on power output ($T_{SH}^{in} = 923$ K, $T_{SH}^{in} = 338$ K, $\varepsilon_R = 0.9$, $\varepsilon_C = \varepsilon_H = 0.7$, $C_C = 1$ kW/K, $\eta_{max} = 35.84\%$, $V_1/V_2 = 3.5$).

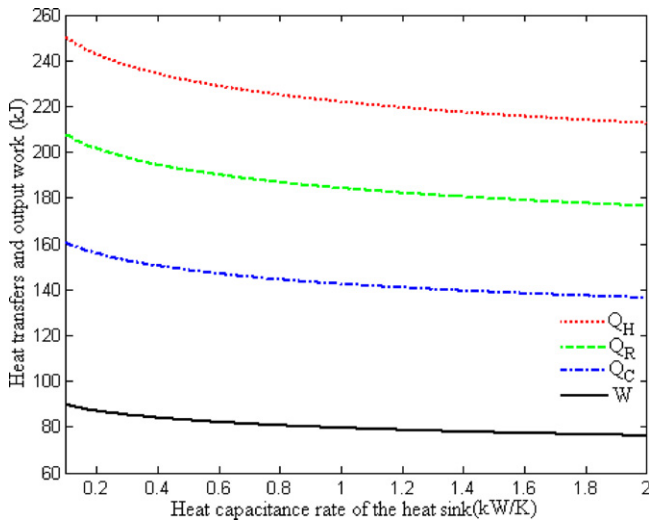


Fig. 16. Effect of the heat capacitance rate of the heat sink on heat transfers and output work ($T_{SH}^{in} = 923\text{ K}$, $T_{SH}^{in} = 338\text{ K}$, $\varepsilon_R = 0.9$, $\varepsilon_C = \varepsilon_H = 0.7$, $C_H = 1\text{ kW/K}$, $\eta_{max} = 35.84\%$, $V_1/V_2 = 3.5$).

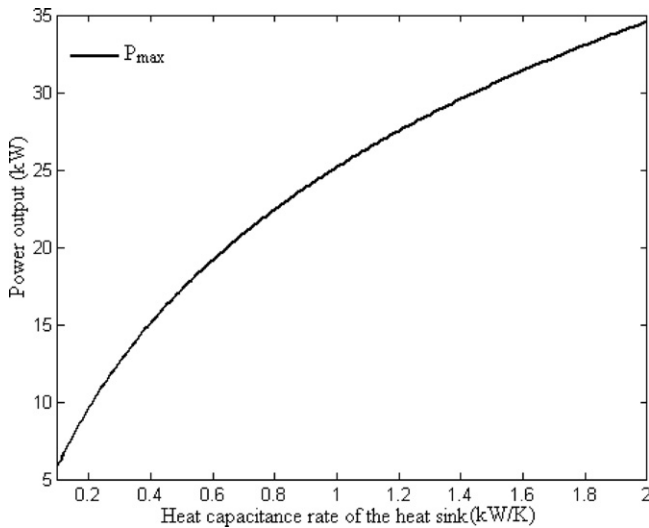


Fig. 17. Effect of the heat capacitance rate of the heat sink on power output ($T_{SH}^{in} = 923\text{ K}$, $T_{SH}^{in} = 338\text{ K}$, $\varepsilon_R = 0.9$, $\varepsilon_C = \varepsilon_H = 0.7$, $C_H = 1\text{ kW/K}$, $\eta_{max} = 35.84\%$, $V_1/V_2 = 3.5$).

5. Conclusion

In a more efficient energy management context, an optimization of an endoreversible Stirling heat engine with finite heat capacitance rates of external fluids in the heat source/sink reservoirs and with regenerative losses is presented and a finite time thermodynamics has been applied to maximize the power output and the corresponding thermal efficiency.

The results showed that the effectiveness of the hot and cold side heat exchangers affect only the maximum power output, while the regenerator effectiveness affects the thermal efficiency only but the external heat source/sink reservoirs fluids inlet temperatures affect both the maximum power output (P_{max}) and the corresponding thermal efficiency. It is also desirable to have a high temperature heat source and low temperature heat sink from the point of view of higher power output (P_{max}) and the corresponding higher thermal efficiency. The volume ratio affect only the maximum power output and in order to obtain maximum output work (W) the constant

volume cooling must be as higher as possible than the constant volume heating. The heat capacitance rates (C_H and C_C) of the external fluids in heat source/sink reservoirs play an important role for higher power output (P_{max}) as well as for lower heat input (Q_H). Increasing the heat capacitance rate (C_H) of the external fluid in the heat source reservoir means increasing the heat input (Q_H) to the heat engine for the same power output. It is also found that increasing the heat capacitance rate (C_C) of the heat sink reservoir (in comparison to C_H) can reduce the heat input (Q_H) to the heat engine for the corresponding same power output. Hence, to obtain higher maximum power output (P_{max}) and lower heat input (Q_H) we must have larger heat capacity of the heat sink in comparison to the heat source reservoir. Thus the present analysis provides a finite time thermodynamic for the design, performance evaluation and improvement of Stirling heat engine.

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